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# CALCULATION OF THE LIBERATION SPECTRUM IN PRODUCTS PRODUCED IN CONTINUOUS MILLING CIRCUITS

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## SUMMARY

A model is proposed to calculate the liberation spectrum that is produced in the progeny from the comminution of a single two-phase particle of arbitrary composition and arbitrary size. This model makes effective use of the modified Andrews-Mika diagram and is based on the simulated breakage of two-phase PARGEN particles. The model is used to calculate the liberation spectrum in all of the streams in a simple milling circuit closed by a hydrocyclone.

## INTRODUCTION

The population balance model is now universally accepted as the appropriate method for the calculation of the particle size distribution that will be produced in a continuously operating milling circuit. This method relies on the so-called breakage-selection function model for the mill and on a model that will describe the classification of a particle population in any classification unit that is incorporated in the closed the milling circuit. These models have been well researched and the general characteristics of the selection function and breakage function are known for most industrially important mill types. This approach to milling circuit analysis is comprehensively described in the excellent text by Austin, Klimpel and Luckie (1984). Extensions to the original models to accommodate autogenous comminution effects have been thoroughly researched particularly by Austin's group and appropriate models are now available and widely used. (Austin 1989).

However, it is not only the particle size distribution of the circuit product that is of importance in practice but, when comminution precedes mineral beneficiation processes, the liberation spectrum is equally, if not more, important. But the calculation of the liberation spectrum that is produced in a continuous closed milling circuit is a vastly more difficult task. No satisfactory and theoretically sound method has yet been developed. This is an important shortcoming of the

population balance approach to the analysis of mineral processing circuits which has in most other respects turned out to be a very sound and successful modelling procedure.

A comparatively simple although approximate method is developed in this paper. This method is imbedded in the standard population balance model for the ball mill and the resulting extended model is shown to be computationally tractable. The method has been incorporated into the popular MODSIM simulation package and thus the product size distribution and liberation spectrum can be calculated in any milling circuit.

The model requires a minimum amount of experimental data but is capable of calculating the detailed liberation spectrum. By contrast other existing methods account only for the completely liberated material and a broad group of particles that contains all incompletely liberated particles. Such a crude description of the liberation spectrum is unlikely to produce models that will have much validity in practice. The method proposed here maintains the fine detail of the complete liberation spectrum but relies on only a single parameter to describe the liberation characteristics of the ore. Such a model can only be approximate and ultimately its usefulness can only be assessed by comparison with measured liberation spectra obtained from operating plants. In spite of the simplification introduced by the use of a single parameter for the description of the liberation characteristics of the ore, the model is based on comprehensive research on the nature of mineral liberation. This model will describe the essential features of the liberation process and will produce results that should be good enough for most practical purposes. The liberation spectrum is used primarily for the calculation of the effect of particle size distribution on leaching processes and also to assess the usefulness of incorporating a regrind mill into any mineral processing circuit particularly flotation circuits. Since the effect of particle composition on leaching and flotation has so far not been precisely modelled, there is not too much point in insisting on highly precise and detailed liberation models. It is all a matter of sensible balance between the need for accuracy and the detail that is actually required to make practically useful calculations.

## A USEFUL LIBERATION MODEL

The mathematical description of the mineral liberation phenomenon is difficult but in spite of the difficulties much progress has already been made on the problem. However, most authors have confined their attention to the prediction of the liberation spectrum when a body of ore is crushed and ground in a batch mill to a particular size distribution. This is not of concern in this paper. We are concerned here with the prediction of the liberation spectrum when particulate material

of a given size distribution and liberation spectrum is comminuted in a continuous mill to a finer size. This is the important practical problem. Austin and Luckie (1986) have presented a review of the state of liberation modelling and the reader unfamiliar with current developments in the field is directed to this excellent paper for the necessary background information.

The fundamental population balance description of the comminution of composite solid materials in a perfectly mixed segment of a ball mill is given by equation 1.

$$p(g,d_p) + S(g,d_p)p(g,d_p)\tau - \tau\int_{R'} b(g,d_p;g',d_p')\,S(g',d_p')\,p(g',d_p')\,dg'\,dd_p' = f(g,d_p) \quad (1)$$

In equation (1) the variables have the following significance. The primary independent variables are the grade (mass fraction of mineral) and size of the particle. These are represented by $g$ and $d_p$ respectively. In this paper we restrict attention to binary ores so that $g$ is a scalar quantity in the range [0,1]. $p(g,d_p)$ is the bivariate distribution density function with respect to the two variables $g$ and $d_p$ in the mill contents. $S(g,d_p)$ is the specific rate of breakage of particles of grade $g$ and size $d_p$. The grade of the particle affects the rate of breakage because the two different mineralogical components will usually not be equally brittle. $f(g,d_p)$ is the bivariate distribution density function that describes the distribution of particle composition and size in the material that is fed to the perfectly mixed zone. $\tau$ is the mean residence time in the perfectly mixed region.

The function $b(g,d_p;g',d_p')$ is our main concern here. It is a bivariate density function in the two variables $g$ and $d_p$ and it describes the distribution of these two variables in the progeny particles that results from the fracture of a particle of grade exactly equal to $g'$ and size equal to $d_p'$. This function cannot be obtained from the various liberation models that describe the liberation spectrum when large lumps of ore are broken since it is necessary to consider breakage of particles having every possible combination of $g'$ and $d_p'$. In principle it is possible to measure $b(g,d_p;g',d_p')$ by direct experimental observation. Particles can be physically sorted into narrow $g'$ and $d_p'$ fractions and many particles from each fraction can be broken and the grade size distributions of the progeny measured. However, such measurements are extremely difficult and tedious and no comprehensive measurements have actually been made. The experiment has however, been simulated by Lin and Cortes (Lin et.al 1988) by simulating the fracture of more or less realistic two-phase particle by digital computer. The results of this simulation are used extensively later in this paper.

An important feature of equation (1) is the restricted region $R'$ in the $g$-$d_p$ space over which the integral operates. The region $R'$ represents every combination of the variables $g'$ and $d_p'$ which can describe parents of a particle that has grade $g$ and size $d_p$ after fracture. The analysis of the shape of the region $R'$ is of great importance in imposing useful restrictions on the form of the function $b(g,d_p;g',d_p')$.

The complexity of the problem can be reduced significantly by decomposing the breakage function as follows

$$b(g,d_p;g',d_p') = b(g|d_p;g',d_p)\,b(d_p;g',d_p') \tag{2}$$

This decomposition effectively decouples the liberation and fracture processes provided that the approximation

$$b(d_p;g',d_p') = b(d_p,d_p') \tag{3}$$

can be made. Equation (3) is true whenever the size distribution of the progeny from a single fracture event is independent of the composition of the parent particle. This is probably quite a good assumption for many ores. In equation (2) $b(g|d_p;g',d_p')$ is the conditional distribution density with respect to the variable $g$ in the subset of particles of size $d_p$ that result from the fracture of particles of composition $g'$ and size $d_p'$.

A particular advantage of the decomposition (2) is that $b(g|d_p;g',d_p')$ is probably largely independent of the energy absorption rate during fracture while $b(d_p;d_p')$ is now known to be a strong function of the specific rate of energy absorption during the fracture event and that depends strongly on the type and operating conditions of the comminution equipment. The determination of $b(g|d_p;g',d_p')$ is very largely a geometrical problem that is governed by the mineralogical texture of the ore.

The essential characteristics of the conditional breakage function were first investigated by Mika and Andrews (1975). Although this pioneering work was restricted to an analysis of mineral liberation during batch milling, their analysis is of the greatest possible importance in tackling the difficult problem of calculating the liberation during continuously operating comminution processes.

Andrews and Mika showed that several important restrictions apply to the form of the breakage function *b*. These restrictions can be understood by reference to figure 1. Consider point A in the particle phase space and consider the fracture of a single particle located in phase space at point A. The progeny particles will reappear at various points in the phase space but not all points in the space are accessible to the progeny. Firstly the size of each daughter fragment ($d_p$) must be less than the size of the parent ($d_p'$) so that $d_p < d_p'$. Further the volume of each phase must be conserved so that the volume of each mineral phase in each daughter fragment must be less than the volume of that phase in the parent particle.

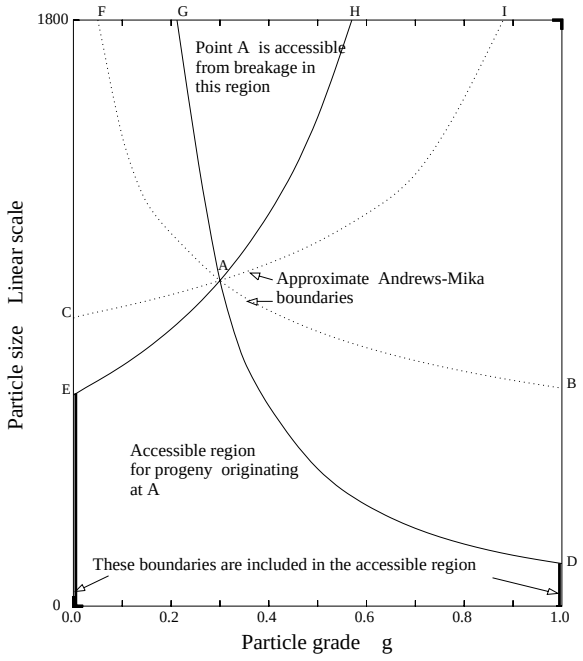


Figure 1 *The accessible region for comminution of single particles. The dotted lines indicate the boundaries originally defined by Andrews and Mika.*

This last condition gives rise to a fuzzy boundary for the accessible region because there can be no unique size $d_p'$ associaed with particles of a particular volume. fortunately the distribution of individual particle volumes about the mean for particles of size $d_p$ is not very high and the fuzziness of the boundary can be neglected for our present purposes. As a useful approximation we consider particle volume equal to $\beta d_p^{\,3}$. Thus the following two inequalities reflect the conservation of phase volume for a binary system

$$g d_p^3 \leq g' d_p'^3 \tag{4}$$

and

$$(1-g) d_p^3 \leq (1-g') d_p'^3 \tag{5}$$

Thes two inequalities are plotted as curves FB and IC in figure 1. The accessible region of the phase space is below AB and AC.

The lines AB and AC are upper bounds for the accessible region and these bounds are tightened if the mineralogical texture of the parent particle is considered. If the largest coherent mineral grain in the parent particle has volume $V_m$ then the volume of the largest possible completely liberated mineral daughter particle is $V_m$. The least upper bounds for the accessible region of a real particle are shown as lines AD and AE in figure 1. The intersection of the lines AD and AE with the axes at $g$=0 and $g$=1 are particularly important. A particle breaking at point A cannot produce completely liberated mineral progeny particles of size greater than $d_{pD}$ and cannot produce completely liberated gangue in progeny particles of size greater than $d_{pE}$. The determination the limiting sizes $d_{pD}$ and $d_{pE}$ is of the greatest importance for the development of predictive equations for the conditional liberation distribution function $b(g|d_p;g',d_p')$.

The only data currently available for the determination of $d_{pD}$ and $d_{pE}$ results from a simulation study conducted at the University of Utah in which simulated two-phase PARGEN particles were subject to random fracture. The products of fracture were classified according to size and mineral content. Since the grade and size of the parent particle were known exactly, the simulation produced the function $p(g,d_p;g',d_p')$ following simulated fracture of sufficient PARGEN particles at each $g'$ and $d_p'$. The results of breaking 500 particles at 3 different values of $g'$ have been published by Lin et.al. (1988). The number of particles examined was not sufficiently high to define the conditional breakage function exactly particularly under conditions when $d_p/d_p'$ is larger than approximately 0.2. However the data presented by Lin et.al. is a representation of the conditional breakage function $b(g|d_p;g',d_p')$ for 6 values of $d_p$. The data show that $b(g|d_p;g',d_p')$ has a characteristic shallow U shape with material shifting further and further out to the liberated edges of the grade spectrum as size decreases. When $d_p$ is not much smaller than the parent size $d_p'$, the distribution shows a peak at values of $g$ close to the parent grade $g'$.

The conditional breakage function must satisfy the consistency conditions

$$\int_0^1 b(g|d_p;g',d_p')\,dg = 1 \qquad (6)$$

and

$$\int_0^1 g\,b(g|d_p;g',d_p)\,dg = g' \qquad (7)$$

at each value of $d_p$ under conditions of random fracture and these equations impose important limitations on the form of the conditional breakage function.

It is important to know the amount of each phase that is completely liberated at each daughter size. This information obtained from the PARGEN simulation is summarised in figure 2. This figure shows the amount of each phase liberated as a function of size. The data is well correlated for all parent particle grades by plotting against the dimensionless variable $d_p S_V^{AB}$ where $d_p$ represents the daughter particle size and $S_V^{AB}$ is the interface area per unit volume of phase A in the parent particle. The data shown in these figures confirm that a fairly well defined intersection with the horizontal axis exists and that this point is independent of parent particle grade at least for the continuous phase. This point represents the size above which no daughter particle can be completely liberated. The simulations at various dispersion densities showed that the limiting size is influenced by the texture of the parent particle as well as its composition. These effects appear to be relatively small however and are neglected in the following treatment.

The interphase area per unit volume of phase is a textural parameter associated with the mineralogical structure of the ore. It can be determined by standard stereol-ogical procedures on polished sections of the ore. As the material is broken the interphase area is conserved provided that fracture is random. At sizes below that at which liberation of the phase is significant a simple argument based on geometrical similitude indicates that the product $d_p S_V^{AB}$ for the parent particle should be invariant with size but should decrease as average particle grade increases.

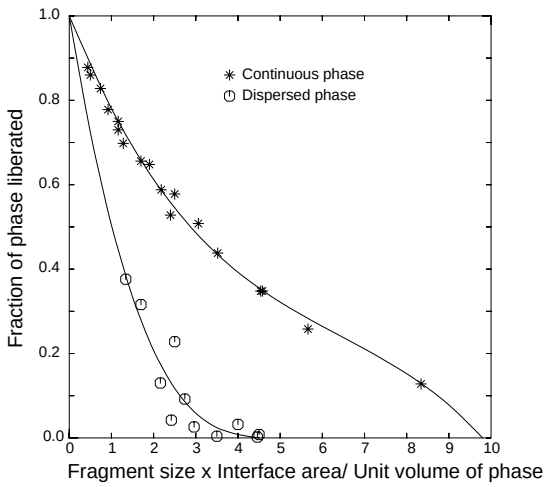


Figure 2 *The fraction of dispersed and continuous phase which is liberated after the breakage of a single multiphase particle. Data from the PARGEN simulation of Lin and Cortes.*

The average values of the group $d_p S_V^{AB}$ for each of the parent PARGEN particles used in the simulations is shown in figure 3. The variation with parent particle grade was found to be linear and the data is well described by

$$d_p S_V^{AB} = \phi_A (1 - g) \tag{8}$$

and

$$d_p S_V^{BA} = \phi_A g \tag{9}$$

where $\phi_A$ is independent of particle grade but is dependent on the mineralogical texture. It is small for course-grained textures and large for fine-grained.

Using the postulate that $d_p S_V^{AB}$ is independent of particle size, it is possible to locate points D and E in figure 1 for any point A using the data from figures 2 and 3. For example consider the case where $g' = 0.3$ as shown in figure 1. From figure 3, $d_p S_V^{AB} = 35$ for the mineral (dispersed) phase and $d_p S_V^{BA} = 15$ for the gangue (continuous) phase. From figure 2 the critical value of $d_p S_V^{AB} = 4.5$ for the mineral phase and critical $d_p S_V^{BA} = 9.8$ for the continuous phase. Thus

$$d_{pD} = \left(\frac{4.5}{35}\right) d_p' = 0.13\, d_p' \tag{10}$$

and

$$d_{pE} = \left(\frac{9.8}{15}\right) d_p' = 0.653\, d_p' \tag{11}$$

At the present time there is no effective procedure for establishing the precise shape of the boundary lines AD and AE.

The example given above indicates that both $d_{pD}$ and $d_{pE}$ are strong functions of the grade of the parent particle. Depending on the value of $g'$ at A, it is possible for the value of $d_{pD}$ or $d_{pE}$ estimated from the critical $d_p S_V^{AB}$ values to lie above $d_{pB}$ and $d_{pC}$ in figure 1. In that case the more restrictive inequality (4) or (5) must be used and lines AB or AC will bound the accessible region.

Just as each point A has associated with it the accessible region that can receive its daughter products, so each point A has the region $R'$ which contains all points in the phase space that include point A in their accessible regions.
Define

$$R_A = \{x;\ A \rightarrow x\} \tag{12}$$

then

$$R_A' = \{x;\ A \in R_x\} \qquad (13)$$

where $x$ is any point in the two-dimensional particle phase space and the symbol $A \rightarrow x$ means that a particle breaking at A can produce progeny at $x$. It is considerably more difficult to establish the boundaries of $R'$ than those of $R_A$. Consequently the computational procedure that we develop will not rely on an explicit knowledge of the boundaries of $R'$.

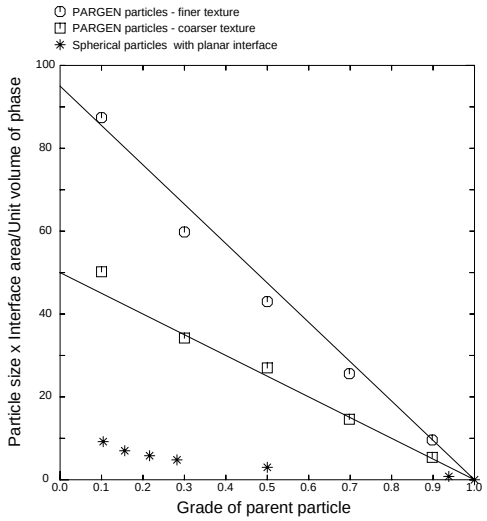


*Figure 3* *Average values of the group $d_p S_V^{AB}$ for the parent PARGEN particles used in the simulations and for spherical 2-phase particles with a planar interface.*

A finite difference analogue to equation (1) is required for practical calculations. Suitable finite difference representations are well known and provide convenient stable computational algorithms. We adopt the notation of Austin and Luckie (1986) and represent equation (1) by

$$p_{ij} = \frac{f_{ij} + \tau \sum_{l=1}^{j-1} b_{jl} \sum_{k=K_{ijl}'}^{K_{ijl}''} S_{kl}\, b_{ijkl}\, p_{kl}}{1 + S_{ij}\tau} \qquad (14)$$

In equation (14) the subscripts $i$ $j$ $k$ and $l$ index the variables $g$, $d_p$, $g'$ and $d_p'$ respectively. Thus

$$\begin{aligned} b_{jl} &= b(d_p; d_p') \\ b_{ijkl} &= b(g \mid d_p'; g', d_P') \\ S_{kl} &= S(g', d_p') \\ p_{ij} &= p(g, d_p) \\ f_{ij} &= f(g, d_p) \end{aligned} \qquad (15)$$

The approximation of equation (3) has been explicitly used so that $b_{jl}$ can be taken out of the summation on $k$. $K'_{ijl}$ and $K''_{ijl}$ define the left and righthand boundaries of $R'$ at size $l$. These boundaries are difficult to locate directly and equation (14) is evaluated using an algorithm that requires explicit location of the boundaries of the accessible region $R_A$ as defined in equation (12).

Let $I'_{jkl}$ and $I''_{jkl}$ be the left and right hand boundaries of region $R_A$ at size $j$. An efficient algorithm for the calculation of the size distribution and liberation spectrum in the products from a perfectly mixed region of a ball mill is given in adjacent box. This algorithm essentially evaluates the rate of breakage in cell $kl$ and allocates the progeny to all cells in the appropriate region $R_A$. This procedure is repeated for every cell $kl$ starting at the largest size. The algorithm is explicit since all progeny must be of smaller size than the parent. The algorithm can accordingly be implemented in a straightforward manner.

```
p_ij := f_ij          for all i and j
For l = 1 to ND-1
    For k = 1 to NG
        Procedure S_kl
        p_kl := p_kl / (1+S_kl)
        For j = l+1 to ND
            Procedure b_jl
            Find I'_jkl to I''_jkl
            For i = I'_jkl to I''_jkl
                Procedure b_ijkl
                p_ij := p_ij + b_ijkl b_jl S_kl p_kl
            End i loop
        End j loop
    End k loop
End l loop
```

1 *An efficient algorithm for the calculation of the liberation spectrum in the products leaving a perfectly mixed segment of a ball mill.*

## THE COMMINUTION AND LIBERATION FUNCTIONS

The mathematical development presented above provides a simple and logical framework for the calculation of the liberation spectrum from a continuously operating ball mill. The algorithm developed is robust and easy to implement by digital computer.

It is clear that it is necessary to develop models for the specific rate of breakage $S_{kl}$, for the comminution breakage function $b_{jl}$ and for the distribution function $b_{ijkl}$. If $S_{kl}$ and $b_{jl}$ are assumed to be independent of $k$ (ie. the specific rate of breakage and the distribution of progeny size are not functions of the composition of the parent particle) then standard comminution experimental procedures and scale-up relationship can be used to calculate these functions. There

is already a large literature associated with modelling of these comminution functions and the book by Austin, Klimpel and Luckie (1984) should be consulted. Of course, if the comminution and liberation phenomena cannot be decoupled then these functions must be determined from comminution experiments on multiphase materials.

Our chief concern here is with the development of a predictive model for the liberation distribution function $b_{ijkl}$. This function has 4 independent variables and is consequently very difficult to model. It is impossible to measure the function experimentally for every possible combination of the four variables although methods using massive parameter estimation procedures have been proposed.

It has been commonly asserted in the literature that the liberation characteristics of an ore can be adequately modelled by classifying the particle composition into only 3 classes - liberated mineral, liberated gangue and all mixed particles lumped together in a single class. Although this reduces the number of parameters very considerably it does not provide the kind of detail that is required to make accurate predictions of the liberation spectrum in the comminution products from which the effect on downstream operations such as leaching and flotation can be calculated. The approach adopted here is to obtain a detailed liberation spectrum covering a large number of composite particle classes and to attempt to find a representation of the liberation distribution function that describes the distribution over many particle classes but which is characterised by a small number of parameters. It turns out that a single parameter representation is possible provided that all of the constraints imposed on the distribution that are described in the previous section are satisfied. The results obtained appear to produce very acceptable results. As yet no experimental data are available against which the model can be tested. It is planned to generate such data in a forthcoming program of research.

The first simplification imposed on the liberation distribution function is that it is a unique function of the ratio $d_{pj}/d_{pl}$ rather than the parent and progeny sizes individually. Thus $b_{ijkl} = b_{ik}(d_{pj}/d_{pl})$. For convenience the arguments in the brackets are omitted in the following so that $b_{ijkl}$ is written $b_{ik}$. The function proposed also varies depending on the position of the daughter fragment in the modified Andrews-Mika diagram. If $d_{pj}$ is larger than both $d_{pE}$ and $d_{pD}$ in figure 1 then the progeny particles have compositions concentrated close to that of the parent particle. Thus for $d_{pj} > d_{pE}$ and $d_{pj} > d_{pD}$, $b_{ik}$ is zero for all $i < k-1$ and $i > k+1$ and

$$b_{k-1\,k} = f_1(1-b_{kk})$$
$$b_{k+1\,k} = f_2(1-b_{kk}) \tag{16}$$

Here $f_1$ and $f_2$ are non-negative fractions.

The finite difference analogues of equations (6) and (7) are

$$b_{k-1\,k} + b_{kk} + b_{k+1\,k} = 1$$
$$b_{k-1\,k}\,g_{k-1} + b_{kk}\,g_k + b_{k+1\,k}\,g_{k+1} = g_k \tag{17}$$

where $g_k$ is the mineral grade in composition class $k$. These have non trivial solutions only if

$$f_1 = \frac{g_{k+1} - g_k}{g_{k+1} - g_{k-1}}$$
$$f_2 = \frac{g_k - g_{k-1}}{g_{k+1} - g_{k-1}} \tag{18}$$

for an arbitrary value of $b_{kk}$ in the range [0,1]. The triangular distribution represented by equations (16) is strongly suggested by the Lin and Cortes PARGEN simulation.

Whenever the ratio $d_{pj}/d_{pl}$ is small enough to allow liberated particles to be produced from the parent, a different model is suggested by the simulation. Thus

$$b_{NG\,k} = L_1 g_k \qquad for \quad d_{pj} < d_{pD} \tag{19}$$

and

$$b_{1k} = L_2(1-g_k) \qquad for \quad d_{pj} < d_{pE} \tag{20}$$

where $L_1$ is the fractional liberation of the mineral (dispersed) phase read from figure 2 and $L_2$ is the fractional liberation of the gangue (continuous) phase read from figure 2. *NG* is the total number of grade classes.

Using the PARGEN simulation as a guide, it is assumed that most of the progeny particles tend to accumulate in the grade classes next to the two liberated extremes and the remainder are distributed uniformly over the interior classes. This is described by

$$b_{ik} = b' \qquad for \quad 3 \le i \le NG-3 \tag{21}$$

The finite difference analogues of equations (6) and (7) now become

$$b_{2k} + b_{NG-1\,k} = 1 - b_{1k} - b_{NG\,k} - (NG-4)b' = a_1 \tag{22}$$

and

$$b_{2k}g_2 + b_{NG-1\,k}g_{NG-1} = g_k - b_{1k}g_1 - b_{NG\,k}g_{NG} - (NG-4)b' \sum_{i=3}^{NG-3} g_i = a_2 \tag{23}$$

The solution to equations (22) and (23) are easily obtained as

$$\begin{aligned} b_{NG-1\,k} &= \frac{a_2 - a_1 g_2}{g_{NG-1} - g_2} \\ b_{2k} &= a_1 - b_{NG-1\,k} \end{aligned} \tag{24}$$

The choice of $b'$ is not entirely arbitrary since a solution must be found to satisfy

$$b_{ik} > 0 \qquad for \quad 1 < i < NG \tag{25}$$

In order to ensure this, the value of $b'$ cannot be too large and a value given by

$$b' = MIN[g_k, 1-g_k]\frac{1-b_{1k}-b_{NG\,k}}{NG-4} \tag{26}$$

has been found to be satisfactory and to produce distributions that are qualitatively similar to these produced in the PARGEN simulation.

Finally we note that the fracture of a completely liberated particle can only produce completely liberated progeny so that

$$\begin{aligned} b_{11} &= b_{NG\,NG} = 1 \\ b_{i1} &= 0 \qquad for \quad i > 1 \\ b_{i\,NG} &= 0 \qquad for \quad i < NG \end{aligned} \tag{27}$$

This completes the specification of the liberation distribution function $b_{ijkl}$ and only 2 arbitrary constants have been used: $\phi_A$ in equations (8) and (9) and $b_{kk}$ in equations (16) and (17).

The calculated liberation spectra from various closed milling circuits was found to be relatively insensitive to $b_{kk}$ and this parameter has been fixed at 0.6 for this study.

## PRACTICAL APPLICATIONS

The model for liberation by comminution of two-phase particles developed in this paper is useful particularly for the calculation of the liberation spectrum of products produced in a continuous closed milling circuit. Most classifiers used in modern mineral processing plants will classify on the basis of both particle size and particle density. For example, the cut point for heavier particles can be considerably smaller than the cut point for lighter particles in a hydrocyclone. The liberation spectrum of the feed to the mill will be a composite of the plant feed and the classifier underflow. Thus to calculate the liberation spectrum in the product stream from the mill a combined liberation and comminution model of the type developed in this paper must be used.

The algorithm described in box 1 has been implemented in Austin's 3-mixed-regions-in-series ball mill model and incorporated in the MODSIM ore dressing plant simulator. This simulator produces the liberation spectrum in any stream in the plant and by way of example a simple single-stage closed ball-milling circuit typical of those used to mill pyritic quartzite on Witwatersrand gold mines has been simulated. The flowsheet is shown in figure 4. The particle size distributions in each stream of the flowsheet are shown in figure 5.

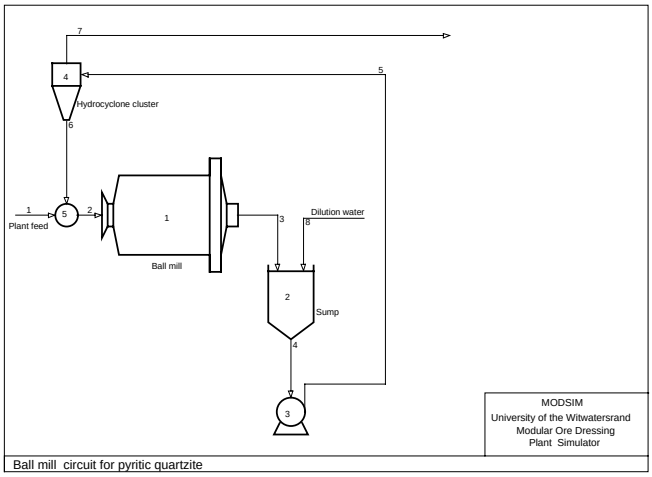


Figure 4 *A typical closed milling circuit used to mill pyritic quartzite. The liberation spectrum in each stream was calculated using the MODSIM simulator.*

The liberation spectra of pyrite in the plant feed, the mill feed, the cyclone underflow and overflow are shown in figures 6 and 7 for two values of the ore texture parameter $\phi_A$. The

effect of the ore texture parameter $\phi_A$ is clearly shown in figures 6 and 7. If the ore texture is characterised by a texture parameter as low as 10 the minerals are complete liberated in the final product stream whereas with $\phi_A$ = 100 only 11% of the pyrite and 61% of the quartite is liberated.

Perhaps the most striking features of the calculated liberation spectra are the two valleys in the distribution function on either end. This reflects the fact that the completely liberated classes act as a sink of material since material can enter those completely liberated classes but never leave them.

This simulation was run without allowing for differential breakage rates between the two minerals although this is easy to allow for in the algorithm. As a result the size distributions produced in the circuit and shown in figure 5 are independent of the ore texture.

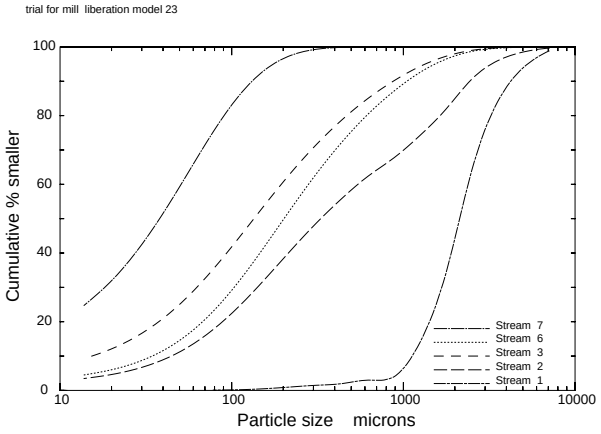


Figure 5 *The particle size distributions in each stream of the milling circuit calculated by the MODSIM simulator.*

## CONCLUSIONS

A model has been developed for the calculation of the liberation spectrum of the progeny that results from the comminution of a single particle of arbitrary composition. The model is based on the results of a computer simulation of single two-phase particle fracture.

Two principles are invoked : Steiner's principle of conservation of interfacial area and Meloy's principle of geometrical similitude. The result is a model with only 2 arbitrary parameters one of which characterises the texture of the ore. This is in marked contrast to other models for the interaction of comminution with mineral liberation which require very many (sometimes hundreds) of parameters. This sparing use of arbitrary parameters makes the model useful for implementation with existing well proven models for comminution such as that of Austin and can easily be implemented in simulation systems such as MODSIM.

It will be comparatively easy to check the predictions of this theory against results obtained in continuously operating milling circuits and this is the subject of laboratory investigations. Estimating the value for the single textural parameter will not be difficult by matching liberation spectra predicted by the model and those measured in operating plants particularly with the

advent of automated stereological procedures now becoming commonplace.

A particular advantage of the model presented here is its ability to cope with a large number particle composition and size classes which is essential for accurate calculation. Existing models, although capable in principle of handling comparatively fine subdivisions of the composition and size coordinates, are in practice limited to at most five composition classes and less than ten size classes because of the enormous number of parameters required. Such a coarse discretization of the particle composition-size space is all but useless for practically effective simulation work.

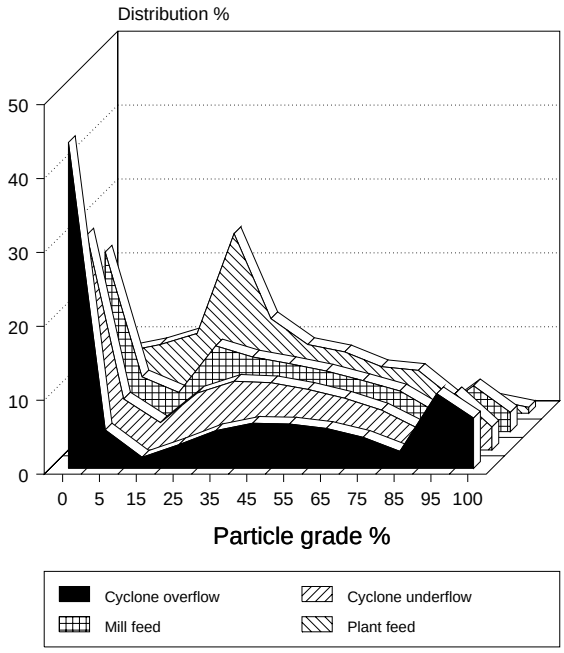


Figure 6 *The calculated liberation spectra in 4 streams in the milling circuit. The mineralogical texture is assumed to be fine with* $\phi_A$*=100.*

Although the advantages of a two-parameter model are very great for practical applications, it would be foolish to conclude that the model in its present over-simplified form will be sufficiently accurate for all purposes. Clearly something more than a single parameter is required to describe the ore texture comprehensively. Furthermore the parameter $\phi_A$ cannot be readily measured directly by image analysis and that is its chief disadvantage. In addition the model is formulated with heavy reliance on the PARGEN simulation experiment particularly through the use of the data in figure 2 to establish the boundaries of the modified Andrews-Mika diagram. The model would benefit from examination of equivalent data obtained in actual laboratory experiments.

In spite of the uncertainties involved with the model formulation, the present work should provide a useful starting point for further investigation of this most important problem. The model has been shown to be tractable for detailed circuit calculations and it makes plausible predictions of the detailed liberation spectra in products from continuous closed circuit mills, the first time that such a calculation has been achieved.

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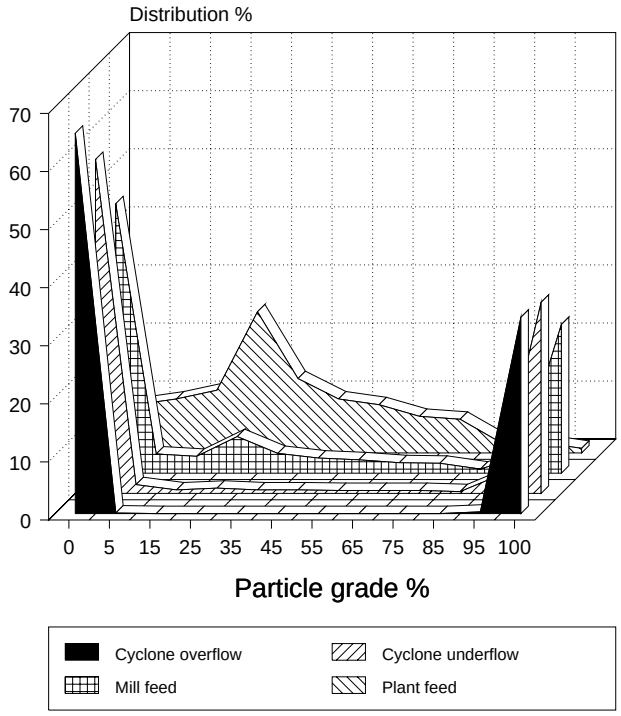


Figure 7 *The calculated liberation spectra. Course-grained ore. Liberation is virtually complete.* $\phi_A$*=10*